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KINETIC ENERGY OF ISOLATED THERMAL VORTICES

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Experiments to study the conditions for thermal-to-kinetic energy transition for an isolated vortex are described. On the basis of laboratory and full-scale measurements it is proved that the individuality of an isolated vortex is characterized by an energy similarity criterion, which is equal to the ratio of the kinetic energy to the potential energy of the vortex.

Systematic experiments on seeding of clouds to decrease the destructive force of hurricanes were begun in 1961, when hurricane Esther in the Atlantic Ocean was seeded with silver iodide crystals for two weeks. Following this, experiments were carried out in hurricanes Bella (1963), Debby (1969), and Ginger (1971). Within the framework of the project "Hurricane Fury," financed by the US government, these experiments were aimed at decreasing the destructive force of storm winds by seeding with silver iodide crystals the clouds that surrounded the "eye of the hurricane." The strategy applied consisted in sampling the heat of phase changes of water from cumulus clouds surrounding the "eye of the hurricane." It was assumed that this would cause expansion of the cloud bank and of the zone of maximum winds. Then, by virtue of the law of the absolute moment of momentum in the form [1]

$$vr = \text{const}$$

the maximum wind velocity must decrease.

However, the results turned out to be unconvincing, since changes in the peripheral velocity v of the wind were not sufficiently large and therefore one could not exclude the possibility of their occurrence due to natural variations encountered in all hurricanes in the process of their motion and interaction with a different relief of the locality.

For the above results to be interpreted correctly, it is necessary to answer the question of what fraction of the latent heat of a hurricane converts to the kinetic energy of rotational motion. Indirect calculations carried out from data on the wind velocity and amount of precipitation showed that this fraction was about 20% [1].

This generated a need for an experimental study of the conditions for transition of the thermal energy of an isolated vortex into the kinetic one. To this end a test facility was constructed for forming an isolated thermal vortex under laboratory conditions (Fig. 1). It consisted of a stand, a cup, and a propeller. The cup, 0.078 m in height, has a conical shape with the larger diameter being equal to 0.2 m. The propeller, 0.18 m in diameter, is made of sheet aluminum $0.8 \cdot 10^{-3}$ m thick. It has ten blades each with a length of 0.07 m and width in the middle equal to 0.018 m. In order to create an additional torque from ascending vapor-air flow, a third of the surface of each blade was bent at an angle of 60^0 to the vertical. The success and accuracy of tests depended on the loss of energy on friction in the bearing which constituted an element of the swirler structure. To minimize friction losses, the properller in our setup was mounted on the shaft of a miniature electric motor. In this case the experimentally determined inertia moment of the propeller $I = 0.501 \cdot 10^{-4}$ kg·m^2.

The tests on the formation of thermal vortices of length l were run as follows. The cup was filled with water, whose volume was determined by the height h (Fig. 1). Then, an electric heater with power N_0 = 900 W was immersed in it. At a temperature of 80^0C and higher the vapor flow evaporating from the surface of water caused rotation of the propeller. However, the start of the process of vortex formation was fixed from the instant of water

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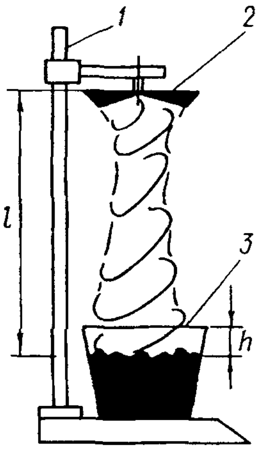


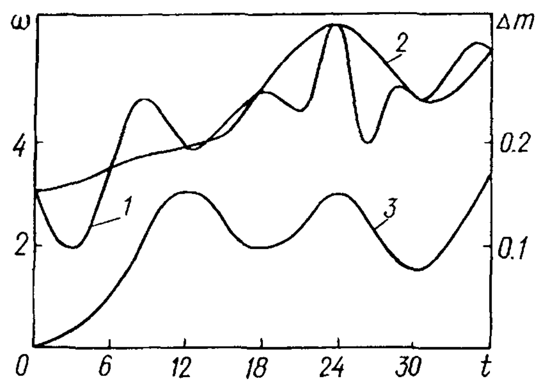


Fig. 1. A setup for simulation of thermal vortices: 1) stand, 2) propeller, 3) cup.

Fig. 2. Time character of the kinematic and energy parameters of vortex: 1) angular velocity of the vortex measured at $\Delta t = 2$ min, 2) same at $\Delta t = 6$ min, 3) mass of water evaporated for $\Delta t = 6$ min.

boiling, since in that case the amount of thermal energy evolved Q could be determined in terms of the mass m of the water evaporated:

$$Q = 2.26 \cdot 10^6 \, m . \tag{1}$$

The process of the separation of vapor flow from the surface of boiling water occurred in separate portions and with a certain periodicity. After separation the portions of vapor acquired translational and rotational motion. The ascending pulsing vapor flow transmitted a torque pulse moment to the propeller, while the translational motion pulse, which acted on the bent portion of the blade, created additional rotation of the propeller. Due to the energy of this additional rotation, the propeller also acted as a swirler of the vapor-air flow. As a result, an isolated thermal vortex was formed which rested on the boiling water surface and the blades of the propeller. The length of the vortex l was determined in the process of tests from the formula

$$l = l_0 + (h - h_0) . \tag{2}$$

Next, using the measured angular velocity ω of the propeller, the kinetic energy of the vortex was calculated from the formula

$$K = \frac{1}{2} I \omega^2 . \tag{3}$$

Observations of separate isolated vortices were made for 30–40 min. It turned out that in this time the vortices did not separate either from the propeller blades, or from the boiling water surface. According to the investigations carried out in [2], such a behavior of vortices means that this setup (Fig. 1) forms stable isolated vortices.

In our experiments the angular velocity ω was calculated from the number of visually fixed revolutions of the propeller for 30 sec. In this case the error of measurements of ω turned out to be equal to 1 rev/min (0.1 $\sec^{-1}$).

To calculate the mass m of the water evaporated, an empirical dependence $m(h)$ was established, where h is the height from the water surface to the upper base of the cup. The maximum error in the measurement of h was $\pm$ 1.5 mm, which, converted to the error of measurement of the mass, is $\pm$ 0.035 kg. It should be added that not more than 30 sec was spent to measure h.

Thus, in constructing the functions $\omega(t)$ and $m(t)$ the error of measurement of the argument t is $\pm$ 1.5 min.

TABLE 1. Character of the Repeatability of Runs Performed on Different Days

t, min	Run 1				Run 2			
	ω, sec^{-1}	l, m	m, kg	N, W	ω, sec^{-1}	l, m	m, kg	N, W
0	3.1	0.475	–	–	4	0.474	–	–
6	3.6	0.477	0.05	314	5.2	0.477	0.07	439
12	3.9	0.483	0.20	628	5.5	0.480	0.14	439
18	5.0	0.487	0.30	628	5.2	0.486	0.29	607
24	6.3	0.493	0.45	706	6.3	0.489	0.37	581
30	5.0	0.496	0.53	665	6.3	0.495	0.52	653
36	5.9	0.503	0.70	732	6.7	0.500	0.65	680

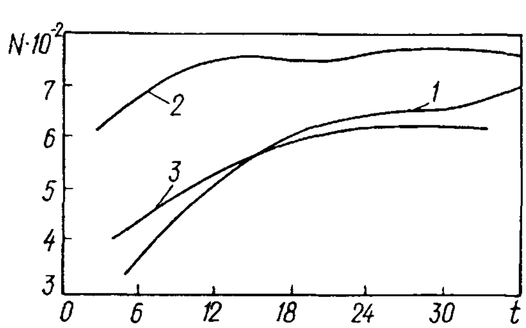
TABLE 2. Parameters of Individual Vortices

t, min	Vortex 1			Vortex 2			Vortex 3		
	ω, sec^{-1}	m, kg	l, m	ω, sec^{-1}	m, kg	l, m	ω, sec^{-1}	m, kg	l, m
0	3.6	–	0.474	4.1	–	0.475	2.6	–	0.475
6	4.4	0.06	0.477	5.3	0.11	0.479	4.3	0.07	0.478
12	4.7	0.17	0.481	6.0	0.24	0.485	4.6	0.17	0.482
18	5.1	0.29	0.486	5.6	0.36	0.489	4.4	0.28	0.487
24	6.3	0.41	0.491	6.3	0.49	0.494	4.5	0.40	0.492
30	5.9	0.52	0.495	7.4	0.62	0.500	3.7	0.50	0.497
36	6.3	0.67	0.501	6.4	0.73	0.504	–	–	–
	$\omega_m = 5.2$			$\omega_m = 5.9$			$\omega_m = 4.0$		

Observation of the rotations of the propeller show that the function $\omega(t)$ has an oscillating character. In view of this, a question arises as to the selection of a time interval Δt in which one can perform reliable measurements with account for the errors indicated. The results of measurements obtained in run 1 and presented in Table 1 are illustrated by Fig. 2. Here, curve 1 reproduces the function $\omega(t)$ from the measurements made at $\Delta t = 2$ min and curve 2 at $\Delta t = 6$ min. From a comparison of these curves it follows that an increase in the interval Δt smoothes out the function $\omega(t)$. However, the above-indicated error of indirect measurements of the mass m for the water evaporated did not allow us in run 1 to carry out measurements of h at $\Delta t = 2$ min. Therefore, the measurements of the height h in this and all the other runs were made at the same time intervals equal to $\Delta t = 6$ min.

Using the values of m listed in Table 1 for run 1, we composed the differences Δm whose numerical values represent the mass of the water evaporated for the time interval $\Delta t = 6$ min. It is seen from this table that the values of Δm obtained in this case exceed the mass measurement error, which is equal to $\pm$ 0.035 kg. Curve 3 in Fig. 2 depicts the function $\Delta m(t)$. The nonmonotonicity of this function indicates that an isolated vortex exerts an influence on the process of water boiling. Starting from a certain instant of time, curves 1 and 3 qualitatively repeat each other with a certain shift along the t axis.

The experiments described were carried out for several months, with the same length l_0 and power N_0 being kept all the time. The curves $\omega(t)$ obtained were diverse in character, but their reproducibility was seen even in the case when two tests were carried out on the same day literally one after the other. Due to the abundance of factors that influence the process of the formation of a stable isolated vortex, there arose the problem of the selection of repeatable vortices from all of the tests carried out. The problem was solved as follows. Using the values of the mass m of the water evaporated at discrete time instants t, we calculated the power



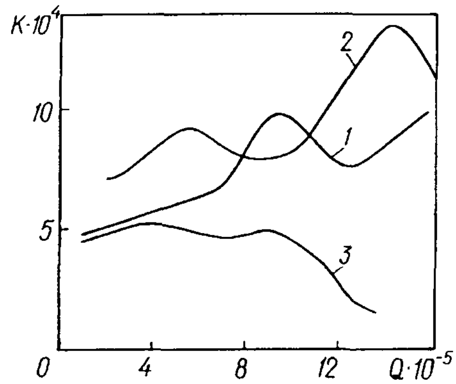


Fig. 3. Power of individual vortices: 1) for vortex 1, 2) 2, 3) 3.

Fig. 4. Kinetic energy of individual vortices: 1) for vortex 1, 2) 2, 3) 3.

$$N = \frac{2.26 \cdot 10^6\, m}{60t}, \tag{4}$$

needed to sustain a vortex. The discrete values of N obtained in this case (see Table 1) were regarded to be experimental and were approximated by the smooth curve $N(t)$. If the smooth curves $N(t)$ coincide in two runs within the error of measurements of the quantities entering into Eq. (4), then such isolated vortices are regarded to be repeatable. Table 1 contains the parameters of two repeatable vortices studied in different days.

We recall that in all of the runs the power N_0 of the electric heater was kept equal to 900 W. This figure exceeds all the values of N given in Table 1. Consequently, a portion of the power $\Delta N = N_0 - N$ is not supplied to the vortex, but is scattered in the surrounding medium. The possibilities provided by the setup did not allow control of the process of scattering of the power ΔN. Precisely this did not allow us to duly reproduce the repeatability of the same vortex.

Two repeatable vortices were used to create a model of an individual vortex by calculating the arithmetic mean values of the corresponding parameters of Table 1. The model obtained in this way was called "Vortex 1" and its characteristics are presented in Table 2. In this case the maximum scatter in the test data from the corresponding mean values amounted to 0.8 sec^{-1} for the angular velocity ω and 0.04 kg for the mass m of the water vaporized.

However, if the scattering of the power ΔN is not controlled during the entire test, the scatter indicated hardly attests to anything. The curve $N(t)$ for individual vortex 1 has a smoothed form (curve 1 in Fig. 3). Two other individual vortices – "Vortex 2" and "Vortex 3" – are obtained in the same way (Table 2). The curves $N(t)$ for the given three vortices (Fig. 3) indicate that the process of the scattering of the power ΔN at the same power N_0 differs quantitatively and qualitatively.

For the individual vortices whose parameters are indicated in Table 2 we calculated the functions $Q(t)$ and $K(t)$ from formulas (1) and (3) and then plotted the curves $K(Q)$ presented in Fig. 4. These curves turned out to be individual, but the function K/Q, depending on time, turned out to be universal for all the vortices (Fig. 5).

Calculations show that the numerical value of K/Q has the order 10^{-9} for the whole time of the existence of an isolated vortex, i.e., only an insignificant fraction of the thermal energy Q supplied to the vortex passes into its kinetic energy of rotational motion. It is for this reason that the earlier mentioned experiments on the seeding of clouds did not lead to the effect desired.

For determining the potential energy of an isolated vortex, the concept of the radial thrust force was introduced in [3]. This force is caused by the pressure gradient and is directed into the interior of the vortex, and its magnitude is determined as

$$F = |\Delta p_m| \pi R^2, \tag{5}$$

where Δp_m is the mean radial pressure difference inside of the vortex and R is its characteristic radius.

TABLE 3. Energy Characteristics of Individual Vortices

t, min	Vortex 1		Vortex 2		Vortex 3	
	$F \cdot 10^3$, N	E	$F \cdot 10^3$, N	E	$F \cdot 10^3$, N	E
0	1.55	0.33	2.12	0.31	0.67	0.39
6	2.50	0.30	3.83	0.29	2.37	0.31
12	2.92	0.30	5.06	0.28	2.77	0.30
18	3.51	0.29	4.33	0.28	2.50	0.30
24	5.64	0.28	5.64	0.28	2.63	0.30
30	4.88	0.28	8.00	0.27	1.66	0.32
36	5.64	0.28	5.82	0.27	–	–
	$E_m = 0.29$		$E_m = 0.28$		$E_m = 0.32$	

Denoting the radial pressure difference on the vortex axis by Δp_0 and the maximum peripheral velocity by v_{max}, the following empirical relations were obtained for them in [3]:

$$|\Delta p_0| = 1.825\,|\Delta p_m|\,, \tag{6}$$

$$|\Delta p_m| = -\,0.823 \cdot 10^{-3}\, v_{max} + 8.35 \cdot 10^{-3}\, v_{max}^2\,. \tag{7}$$

The pressure in (6)-(7) was measured in millibars and the velocity in meters per second. These formulas must be augmented by a formula for the pressure at the center of the vortex p_0:

$$p_0 = 1000 \text{ m bar} - |\Delta p_0|\,. \tag{8}$$

From this it follows that in order to determine $|\Delta p_m|$ it is necessary to know either the pressure at the center of the vortex p_0 or the maximum peripheral velocity v_{max}.

Relations (6)-(8) were determined from an analysis of the data obtained in modeling an isolated vortex under laboratory conditions. For this case, the Atkinson–Holliday formula is available [4]:

$$v_{max} = 3.44 \cdot (1010 - p_0)^{0.644}\,, \tag{9}$$

which establishes a relationship between the pressure p_0 at the center of a hurricane and its maximum velocity v_{max}. This formula is also empirical, but it was obtained from an analysis of measurements carried out in actual circumstances in hurricanes and typhoons. Calculations performed in [5] show that there is a satisfactory quantitative correspondence between formulas (6)-(8) and (9).

To calculate the radial thrust force in application to the data of Table 2 we adopted that $R = 0.09$ m (the radius of the propeller) and $v_{max} = R\omega$. Corresponding calculations by Eq. (5) and (7) are presented in Table 3.

It is quite obvious that the thrust force of a vortex related to the area represents its potential energy. Let us introduce the energetic parameter

$$E = \frac{\rho v_{max}^2 R^2}{F}\,, \tag{10}$$

which is equal to the ratio of the kinetic energy of the vortex to its potential energy. If this criterion remains constant for the entire period of the existence of an isolated vortex, then the number E can be adopted to be the similarity number.

TABLE 4. Parameters of Hurricane David

Date	v_{max}, m/sec	$R \cdot 10^{-4}$, m	$F \cdot 10^{-13}$, N	E	Damage in dollars
29.08	66	8.5	8.3	0.49	$5 \cdot 10^9$
30.08	71	8.3	9.0	0.50	$5 \cdot 10^8$
31.08	70	8.3	8.8	0.50	$5 \cdot 10^9$
02.09	37	10.1	3.8	0.47	without damage
03.09	43	9.7	4.7	0.48	$5 \cdot 10^5$
04.09	44	9.6	5.0	0.47	–
05.09	32	10.6	3.2	0.46	–

TABLE 5. Parameters Characterizing the Effect on Hurricane Debby

No. of seeding	v_{max}, m/sec	$R \cdot 10^{-4}$, m	$F \cdot 10^{-13}$, N	E
0	49	9.3	5.7	0.48
3	44	9.6	4.9	0.48
5	38	10.0	3.9	0.39

To calculate the number E from the data of Table 3, the vapor density ρ was taken to be equal to 0.5977 kg/m^3. Table 3 shows that for each of the individual vortices considered the number E remains constant for the period of their existence within the error of measurements of the quantities entering into Eq. (10).

The question arises as to the accuracy (decimal place) to which E is to be calculated for discerning the individuality of an isolated vortex. This problem was not investigated in detail, but a comparison of the arithmetic mean values of E_m from Table 3 with the curves $N(t)$ in Fig. 3 allowed us to limit ourselves to two decimal places for E.

We recall that on our setup (Fig. 1) an isolated vortex was created by supplying thermal energy Q. The numerical values of K/Q obtained in this case (Fig. 5) turned out to be equal to 10^{-9}, i.e., the efficiency of such a heat engine is not large. At the same time the numerical values of E (Table 3) testify to the fact that if an isolated vortex is created by the formation of potential energy, then the efficiency increases sharply.

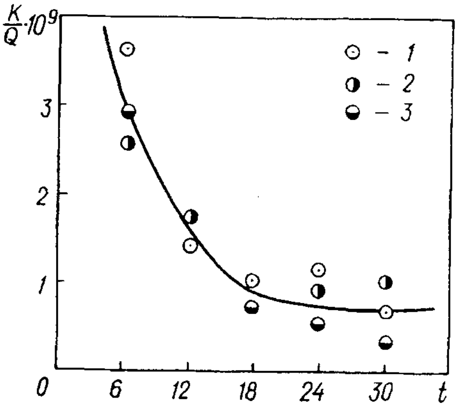
For more convincing evidence that the energy parameter E is a factor of the individuality of an isolated vortex, we will consider the actual data for hurricane David.

The North-Atlantic hurricane David existed from August 25 to September 07, 1979. Its parameters and the damage caused by it are presented in [6, 7]. The results of actual measurements of hurricane David are processed in [5]. A portion of the results borrowed from [5] is presented in Table 4. Here R is the hurricane radius at which the axial projection ω_z of the velocity vector curl vanishes after the last maximum of the peripheral velocity.

It should be noted that no data were presented in [6, 7] for the radial profiles of the peripheral velocity of hurricane David in application to all of the days of its existence. Therefore, the values of R were calculated from the peripheral velocity profiles borrowed from [8] for four tropical cyclones: Helen, Daisy, Ginger, and Esther. The functions ω_z for them, depending on the radial coordinate, had both positive and negative regions of values. The calculated values of R for the cyclones indicated made it possible to establish the following empirical relation:

$$R = \frac{3.1 \cdot 10^5}{v_{max}^{0.31}}, \tag{11}$$

where R is measured in meters and v_{max} in meters per second. Formula (11) allowed us to calculate the radii of hurricane David in all of the days of its existence (see Table 4). Of interest is the behavior of the radial thrust



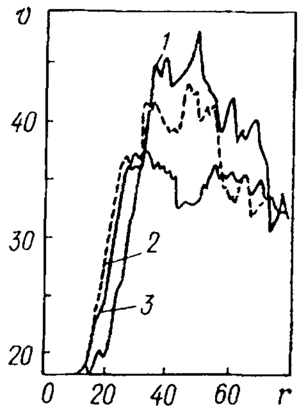


Fig. 5. A universal energy function for individual vortices: 1) for vortex 1, 2) 2, 3) 3.

Fig. 6. Effect of cloud seeding on peripheral velocity profile of hurricane Debby: 1) before first seeding, 2) after third seeding, 3) in 4*h* after fifth seeding. v, m/sec; r, km.

force F depending on the day of existence of the hurricane. For example, on the 2nd of September hurricane David did not inflict any damage and its force F turned out to be minimal. The state of an isolated vortex in which its potential energy is minimal is called unstable in [2]. In spite of the fact that hurricane David existed in both stable and unstable states, the criterion E was held constant (see Table 4) during all of the days. It should be added that for E calculated from Eq. (10) the air density ρ was assumed to be equal to 1.293 kg/m^3.

Let us return again to Table 4 and note that on the 28th of August hurricane David inflicted its maximum damage and its radial thrust force was $F_1 = 8.3 \cdot 10^{13}$ N. According to the classification given in [2], this state of an isolated vortex should be called stable. After that, on two subsequent days it had nearly the same energy state. But on the 2nd of September hurricane David did not cause any damage, its peripheral velocity decreased sharply, and the thrust force turned out to be equal to $3.8 \cdot 10^{13}$ N. For the numerical values of the radial thrust force indicated, the ratio $F_0/F_1 = 0.46$, i.e., the potential energy of the hurricane in unstable state constitutes 46% of the potential energy of its stable state.

The latter result corresponds to the experimental results obtained in [2] under laboratory conditions for the energy characteristics of an isolated vortex in stable and unstable states.

Now, we shall analyze the change in the potential energy on seeding of clouds with foreign particles using as an example hurricane Debby, for which Fig. 6, borrowed from [8], illustrates the character of the change in the radial profile of the peripheral velocity at an altitude of 3660 m caused by a series of seedings on the 18th of August 1969. We note that the experiment with this hurricane is considered to be most successful. Here, 4 hours after the fifth seeding there was really a considerable decrease in the peripheral wind velocity (curve 3 in Fig. 6).

According to the procedure presented above, using the values of v_{max} that correspond to each curve of Fig. 6, we carried out a calculation of the radius of hurricane Debby, its thrust force, and the criterion E (Table 5). It turned out that after the fifth seeding the potential energy of hurricane Debby constituted 69% of its energy before the first seeding. The earlier analysis of hurricane David indicates that the cloud seeding did not transfer the hurricane into unstable state. The reason for this was discovered above, i.e., the change in the thermal energy of the vortex exerts a weak influence on its kinetic energy of rotational motion.

NOTATION

v, peripheral velocity; r, current radial coordinate; t, time; R, radius of vortex; l, length of vortex; h_0, initial height from the upper base of cup to surface of water; l_0, distance from propeller to water surface at the beginning of a run; Q, thermal energy; K, kinetic energy of rotational motion; F, radial thrust force of vortex; E, similarity number, equal to the ratio of the kinetic energy of the vortex to its potential energy; ω_m, mean angular velocity of vortex.

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